التكامل

- قوانين التكامل :

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad : n \neq -1$$

$$\int \frac{dx}{x} = \ln x + c \quad : n = -1$$

$$\int a^x . dx = \frac{a^x}{\ln a} + c$$

$$\int e^x dx = e^x + c$$

$$\int \sin x \, dx = -\cos x + c$$

$$\int \cos x \, dx = \sin x + c$$

$$\int \frac{dx}{\cos^2 x} = \tan x + c$$

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$$\int \frac{dx}{\sin^2 x} = -\cot x + C$$

$$\int \sh x = \ch x + C$$

$$\int \ch x = \sh x + C$$

$$\int \ln x \, dx = x \ln x - x + C$$

$$\int \frac{dx}{ax+b} = \frac{1}{a} \ln|ax+b| + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + C$$

$$\int \frac{dx}{x^2-a^2} = \int \frac{dx}{(x-a)(x+a)} = \frac{A}{x-a} + \frac{B}{x+a}$$

التذكرة :

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

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Ex1 : $\int \left(\cos^2 x - 3^x + \sqrt[3]{x} + \frac{5}{x+1}\right) dx =$

$\int \left(\frac{1}{2} + \frac{\cos 2x}{2} - 3^x + x^{\frac{1}{3}} + \frac{5}{x^2+1}\right) dx =$

$= \frac{1}{2}x + \frac{1}{4}\sin 2x - \frac{3^x}{\ln 3} + \frac{3}{4}x^{\frac{4}{3}} + 5\tan^{-1} x + C$

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Ex2 : $\int \left(\sin^2 mx + \frac{x^2}{\sqrt{x}}\right) dx$

$= \int \left(\frac{1}{2} - \frac{1}{2}\cos 2mx + x^{\frac{3}{2}}\right) dx$

$= \frac{1}{2}x - \frac{1}{4m}\sin 2mx + \frac{2}{5}x^{\frac{5}{2}} + C$

Ex3 : $\int (3^x \cdot 5^{2x}) dx = \int (3^x \cdot 25^x) dx =$

$= \int 75^x dx = \frac{75^x}{\ln 75} + C$

Ex4 : $\int \left(\sqrt{x} + \frac{1}{\sqrt[3]{x}}\right)^2 dx$

$= \int \left(x + 2\frac{x^{\frac{3}{6}}}{x^{\frac{2}{6}}} + x^{-\frac{2}{3}}\right) dx =$

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$$= \int \left(x + 2x^{\frac{1}{6}} + x^{-\frac{1}{3}}\right) dx$$

$$= \frac{x^2}{2} + \frac{12}{7} x^{\frac{7}{6}} + 3x^{\frac{1}{3}} + C$$

Ex 5 : $\int (\tan x + \cot x)^2 dx$

$$= \int (\tan^2 x + 2\tan x \cot x + \cot^2 x) dx$$

$$= \int (\tan^2 x + 1 + \cot^2 x + 1) dx$$

$$= \tan x - \cot x + C$$

ideas...

$$\cot^2 x + 1 = \frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x}$$

Ex 6 : $\int \sqrt{1+x^2} \cdot x^5 dx$

$x^5 = x \cdot x^4$

$t = 1 + x^2 \Rightarrow dt = 2x\,dx$

$x^2 = t - 1$

$x^4 = t^2 - 2t + 1$

$$= \frac{1}{2} \int t^{\frac{1}{2}} (t^2 - 2t + 1) dt$$

$$= \frac{1}{2} \int \left(t^{\frac{5}{2}} - 2t^{\frac{3}{2}} + t^{\frac{1}{2}}\right) dt$$

$$= \frac{1}{2} \left(\frac{2}{7} t^{\frac{7}{2}} - \frac{4}{5} t^{\frac{5}{4}} + \frac{2}{3} t^{\frac{3}{2}} + C\right)$$

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Ex 1: $\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$

$t = \sqrt{x} \Rightarrow dt = \frac{1}{2\sqrt{x}}\,dx$

$= 2\int \sin t\,dt$

$= -2\cos t + C$

$= -2\cos\sqrt{x} + C$

انتهت المحاضرة

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